rectangled Cone whofe Altitude is equal to *A R*. And its Curve Surface will be equal to half the Surface of a Globe whofe Radius is *A R*. So that if the Curve be continued *both ways* infinitely (as its Nature requires) the whole Surface will be equal to that of a Globe of the fame Radius *A R*.

The Defcription of the Rular and Wheel, *Fig.* 2, is fufficient for the Demonftration of the Properties of the Curve : but in order to an actual Conftruction for Ufe, I have added *Fig.* 5. where *A B* is a brafs Rular ; *w h* the little Wheel, which muft be made to move freely and tight upon its Axe (like a Watch-Wheel) the Axe being exactly perpendicular to the Edge of the Rular. *s* reprefents a little Screw-pin to fet at feveral Diftances for different Radii, and its under End is to flide by the Edge of the other fixt Rular. *p* is a Stud for convenient holding the Rular in its Motion.

Note, *Moft of thefe Properties of this* Curve *by the Name of* la Tractrice, *are to be found in a* Memoire *of M.* Bomie *among thofe of the* Royal Academy *of* Sciences *for the Year* 1712, *but not publifh'd till* 1715 : *Whereas this Paper of Mr.* Perks *was produced before the* Royal Society *in* May 1714, *as appears by their Journal.*

VI. *An Account of a Book entituled* Methodus Incrementorum, *Auctore* Brook Taylor, LL,D. & R. S. Secr. *By the Author.*

W HEN I apply'd my felf to confider throughly the Nature of the Method of Fluxions, which has juftly been the Occafion of fo much Glory to its great Inventor Sir *Ifaac Newton* our moft worthy Prefident, I fell by degrees into the Method of Increments, which I have endeavour'd to explain in this Treatife. For it being the Foundation of the Method of Fluxions that the Flux-

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ions of Quantities are proportional to the nafcent Incre-
ments of thofe Quantities: in order to underftand that Me-
thod throughly, I found it neceffary to confider well the
Properties of Increments in general. And from thofe
Properties I faw it would be eafy to draw a perfect Know-
ledge of the Method of Fluxions : for if in any cafe the
Increments are fuppofed to vanifh and to become equal to
nothing, their Proportions become immediately the fame
with the Proportions of the Fluxions. In this Method I
confider Quantities, as formed by a continual Addition
of parts of a finite Magnitude, and thofe parts I call the
Increments of the Quantities they belong to, becaufe
that by the Addition of them the Quantities are increa-
fed. Thefe parts being confider'd as formed in the fame
manner by a continual Addition of other parts ; thence
follows the Confideration of fecond Increments ; and fo
on to third, fourth, and other Increments of a higher
kind. For Example, if x ftands for any Number in the
Series o. 1. 4. 10. 20. 35, &c. in which the Numbers are
formed by a continual Addition of the Numbers in the
Series 1. 3. 6. 10. 15, &c. then the Numbers in the lat-
ter Series are call'd the Increments of the Numbers in the
foregoing Series ; thus, for Example, if to the third
Number (4) in the firft Series, I add the correfponding
third Number (6) in the fecond Series, I fhall produce the
next, that is the fourth Number (10) in the firft Series,
and fo the reft. Any Number in the firft Series being
call'd x, the correfponding Number (which is its Incre-
ment) in the fecond Series I exprefs by $\dot{x}$. And thefe Num-
bers $\dot{x}$ being form'd in the fame manner by the Numbers
in the Series 1. 2. 3. 4, 5, &c. I call thefe laft Numbers $\ddot{x}$,
they being the firft Increments of the Numbers $\dot{x}$, and the
fecond Increments of the Numbers x; and fo on. Hence
having given any Series of Numbers that are call'd by a
general Character x, their Increments are found by taking

their

their Differences; thus in the prefent Example, the firft
Increments *x* in the Series 1. 3. 6. 10. 15, &c. are found by
taking the Differences of the Numbers *x* in the Series 1.
4. 10. 20. 35, &c. and the fecond Increments *x* in the
Series 1. 2. 3. 4. 5, &c. are found in the like manner, by
taking the Differences of the Numbers *x*, and fo of the
third and other Increments. This Method confifts of two
parts; One is concerned in fhewing how to find the Re-
lations of the Increments of feveral variable Quantities,
having given the Relations of the Quantities themfelves;
and the Other is concerned in finding the Relations of the
Integral Quantities themfelves freed from the confidera-
tion of their Increments, having given the Relations of
the Increments: either fimply, or they being any how
compounded with their Integral Quantities. In the Me-
thod of Fluxions Quantities are not confider'd with their
parts, but with the Velocities of the Motions they are
fuppofed to be formed by; or to fpeak more accurately,
they are confider'd with the Quantities of the Motions by
which they are fuppofed to be generated; for the Fluxions
are proportional to the Velocities, only when the moving
Quantities, which produce the flowing Quantities confi-
der'd, are equal. Thefe Quantities of Motion, or Velo-
cities when the moving Quantities are equal, are what
Sir *Ifaac Newton* calls Fluxions. As in the Method of
Increments there are fecond, third, and other Increments;
fo in the Method of Fluxions there are fecond, third, and
other Fluxions; the Fluxions themfelves being confider'd
as Quantities that are formed by Motion, the Quantity
of which Motion is their Fluxions. As the Method of
Increments confifts of two Parts; one being concern'd in
finding the Increments from the Integrals given, and the
other in finding the Integrals having given the Increments;
fo does the Method of Fluxions confift of two Parts;
the one fhewing how to find the Fluxions, having the

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the Fluents given ; and the other fhewing how to find the
Fluents freed from Fluxions, having given the Relations
of the Fluxions, whether compounded with their Fluents
or otherwife. The Principles of this Method may all be
drawn directly as a Corollary from the Principles of the
Method of Increments. For Sir *Ifaac Newton* having de-
monftrated (*Phil. Nat. Princ. Math. Sect.* 1. and in the
Beginning of his Treatife *De Quadratura Curvarum*) that
the Fluxions of Quantities are proportional to their naf-
cent or evanafcent Increments, if in any Propofition rela-
ting to Increments, you make the Increments to vanifh
and to become equal to nothing, and for their Proportion
put the Fluxions, you will have a Propofition that will
be true in the Method of Fluxions. This is but a Corol-
lary to Sir *Ifaac Newton*'s Demonftration of the Fluxions
being proportional to the nafcent Increments. For this
reafon, to make the Method of Fluxions to be underftood
more throughly, I thought it proper to treat of thefe two
Methods together, and I have handled them promifcu-
oufly as if they were but one Method. Some people, be-
caufe that the Fluxions are proportional to the nafcent
Increments of Quantities, have thought that by the Me-
thod of Fluxions Sir *Ifaac Newton* has introduced into Ma-
thematicks rhe Confideration of infinitely littleQuantities;
as if there were any fuch thing as a real Quantity infinite-
ly little. But in this they are miftaken, for Sir *Ifaac*
does only confider the firft or laft Ratio's of Quantities,
when they begin to be, or when they vanifh, not after they
are become fomething, or juft before they vanifh ; but in
the very moment when they do fo. In this cafe Quantities
are not confider'd as infinitely little ; but they are really
nothing at the time that Sir *Ifaac* takes the Proportions
of their Fluxions ; and the Truth of this Method is
demonftrated from the Principles of the Method of In-
crements, in the fame manner as the Ancients demonftra-
ted

ted their Conclufions in the Method of Exhauftions, by a *Deductio ad Abfurdum*.

Having premifed thus much in general concerning the two Methods here treated of, to come to a particular Defcription of this Book; In the Preface I give a fhort Defcription of the Method of Increments, and an Account of Sir *Ifaac Newton*'s Notion of the Fluxions which I have already fpoke of. The Book confifts of two Parts, and contains 118 Pages in *Quarto*; the Propofitions being number'd throughout from the Beginning. In the firft Part I explain the Principles of both Methods : and in the fecond Part I fhew the Ufefulnefs of them in fome particular Examples.

After having explain'd the Notation I make ufe of in the Introduction, in the firft Propofition I explain the direct Method both of Increments and of Fluxions. The fecond Propofition fhews how to transform an Equation wherein Integrals and their Increments, or wherein Fluents and their Fluxions are concern'd ; fo as in the Room of the Integrals or Fluents, to fubftitute their Compliments to a given Quantity with their Increments or their Fluxions, they increafing in a contrary Senfe to the Quantities in the firft Suppofition. In the third Propofition I fhew how to transform a fluxional Equation, fo as to change the Characters of the Fluents, making that Quantity to flow uniformly which in the firft Suppofition flow'd unequally, having fecond, third and other Fluxions, and making that Quantity which in the firft Suppofition flow'd uniformly, now to flow unequally, fo as to have fecond and third Fluxions, &c. This Propofition is of great ufe in the inverfe Method, when you would invert the Expreffion of the Relation of the flowing Quantities ; for Example, if in the Suppofition *z* flows uniformly and *x* variably, by the inverfe Method of Fluxions you will find *x* expreft by the Powers of *z* : but if you would find *z*

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exprest by the Powers of x, you muft then transform the Equation by this Propofition. Sir *Ifaac Newton* and Mr *de Moivre* do this by the Reverfion of Seriefes; but I take this to be the more proper and more genuine Method of doing it directly. In the fourth and fifth Propofitions are explain'd the Method of judging of the Nature and Number of the Conditions that may accompany an *Inctemental* or a Fluxional Equation. This is a Circumftance that I don't find to have been explain'd by any one before, and the Propofitions are fomething intricate; wherefore it will not be improper to explain this Matter a little more at large. Suppofe then that z and x are two variable Quantities, and fuppofing z to increafe uniformly by the continual Addition of its conftant Increment z, (according to the Notation I make ufe of in this Book) fuppofe $x + z = x$. Then if it be propofed as a Problem to find the Value of x, exprefs'd by the Powers of z, and quite freed from the Increments; by the fourth Propofition there may be three Conditions added to this Problem. The Demonftration of this is taken from the Formation of the Integrals by a continual Addition of their Increments. Suppofe that all the Values of z, x, x, x, x, &c. were fet down in order in fo many Columns, and that at the Head of the Table, the correfponding Values of z, x, x, x, x, were exprest by the Symbols a, c, c, c, c. Then by means of the given Equation $x + z = x$, or $x = x - z$, will $c = c - a$, whence the fecond correfponding Values of z, x, x, x, x will be $a + z$, $c + c$, $c + c$, $c + c - a (= c + c)$ and $c + c - a - z$ (by the Eq.) Whence again the third Values are $a + 2z$, $c + 2c + c$, $c + 2c + c - a (= c + 2c + c)$ $c + 2c - 2a + c - z$, and $c + 2c + c - a - 2z$, and fo you may proceed to find the fourth, fifth, and all

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the other Values of $z, x, \overset{..}{x}, \overset{...}{x}, \overset{....}{x}$. But by what is already set down, it is evident that all their Values will be expreſt by a and z, and the three Symbols c, c, c : and conſequently all the Values of the reſt of the Increments of x, viz. $\overset{.}{x}, \overset{..}{x}, \&c.$ will be expreſs'd by the ſame Symbols. Whence it follows that to determine the Values of the Symbols c, c, c, there may be taken three Conditions relating to the Values of $x, \overset{.}{x}, \overset{..}{x}, \overset{...}{x}, \overset{....}{x}, \&c.$ promiſcuouſly, as the fourth Propoſition directs. The ſame Rule holds good in the Method of Fluxions. For Example, having given the Æquation $a\,z^2 + x^2 \overset{..}{x} = 0$, if it be propoſed to deſcribe the Curve that it belongs to; by the fourth Propoſition it may alſo be required as a Condition, that the Curve ſhall paſs through two given Points; that it ſhall touch two given Lines; that it ſhall paſs thro' a given Point, and when it cuts a given Line ſhall have a given Degree of Curvature; or that it ſhall have any other two Circumſtances that depend upon the Values of the third, fourth or other Fluxions. Theſe Conditions that attend Incremental or Fluxional Equations, I don't know to have been ſufficiently taken notice of by any Body: but they ought well to be attended to in the Inverſe Methods; the Solutions of particular Problems being never perfect, unleſs there be proviſion made for the ſatisfying of them, by the indetermined Coefficients in the Equation that contains the Solution of the Problem. Examples of this may be ſeen in Prop. 17 and 18, where I give the Solution of the Problems concerning the *Iſoperimeter*, and the *Catenaria*.

The ſixth Propoſition contains the general Explanation of the Inverſe Method both of Fluxions and of Increments, which conſiſts in the Solution of this Problem. Having given the Relations of the Increments, or of the Fluxions of ſeveral Quantities, whether they be conſide-

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red with their proper Integrals or with their proper
Fluents or not ; to find the Relations of the Integrals or
of the Fluents, freed from their Increments or from their
Fluxions. The Direction I have given for finding the So-
lution in finite Terms is but tentative. And I muft con-
fefs I know of no other Method that is general for all Ca-
fes. For I can find no certain Rule to judge in general,
whether any propofed Equation, involving Increments
or involving Fluxions, can be refolved in finite Terms.
For this Reafon we are obliged to feek the general Solu-
tion in infinite Seriefes ; which when they break off, or
when they can any way be reduced to finite Terms, they
then contain the Solutions which we always hope for.
The Method of finding thefe Seriefes is explain'd in the
eighth Propofition, and that is by means of a Series that
is demonftrated in the feventh Propofition. And this I
take to be the only genuine and general Solution of the
inverfe Methods. For in this Solution you always have
thofe indetermined Coefficients, which are neceffary to
adapt the Equation that is found to the Conditions of the
Problem propofed. For want of this Circumftance all
other Methods are imperfect ; and particularly Sir *Ifaac
Newton*'s Method of finding Seriefes by a Rular and Pa-
rallelograms labours under this Difficulty, becaufe it
brings no new Coefficients into the refulting Equation,
which may afterwards be determined by the Conditions
of the Problem. However becaufe this Method is very
ingenious and very elegant, I thought it proper to ex-
plain it in the following (*viz.* the 9*th*) Prop. The 10*th*.
11*th*, and 12*th* Propofitions conclude the firft Part, and
in them I treat of the manner of finding the Integral or
the Fluent, having given the Expreffion of a particular
Increment, or of a particular Fluxion of it ; without be-
ing involved with the Integrals, or with the Fluents, or
with any other Increments, or with any other Fluxions of

it. This is a particular Case of the Inverse Method, but for its great usefulness I thought it deserved particularly to be taken notice of. This Problem is treated of in general in the 10th Proposition. The Method of solving it in finite Terms is only tentative ; and when that does not succeed, recourse must necessarily be had to the Solution by a Series in the 8th Proposition. In the 11th and 12th Propositions I have shew'd how Seriefes may be conveniently found, in some particular Cases when Fluxions are propofed.

In the second Part I have endeavour'd to shew the Usefulness of these Methods in the Solution of several Problems; The 13th Proposition is much the same with Sir *Ifaac Newton*'s *Methodus Differentialis*, when the Ordinates are at equal Diftances : and in an Example at the End of this Proposition I have shew'd how easily Sir *Ifaac Newton*'s Series for expreffing the Dignity of a Binomial may be found by this Incremental Method. The 14th Proposition shews in some meafure how this Method may be of ufe in summing up of Arithmetical Seriefes. In the 15th Proposition I shew by some Examples how the Proportions of the Fluxions are to be found in Geometrical Figures ; from whence immediately flows the Method of finding the Radiufes of their inofculating Circles, the Invention of the Points of contrary Flexure, and the Solution of other Problems of the like nature. In the 16th Proposition I shew how the Method of Fluxions is to be applied to the Quadrature of all forts of Curves. In the following Proposition I give a general Solution of the Problem of the *Ifoperimeter*, which has been treated of by the two famous Mathematical Brothers the *Bern ulli*'s. In the 18th Proposition I give the Solution of the Problem about the *Catenaria*, not only when the Chain is of a given Thickness every where, but in general, when its Thickness alters according to any given Law. In the

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following Propofition I fhew the *Fornix* or Arch which fupports its own Weight to be the fame with the *Catenaria*. In the two next Propofitions I fhew how to find the Figures of pliable Surfaces which are charged with the Weight of a Fluid. In the 22*d* and 23*d* Propofitions I treat of the Motion of a Mufical String, and give the Solution of this Problem: To find the Number of Vibrations that a String will make in a certain time, having given its Length, its Weight, and the Weight that ſtretches it. This Problem I take to be entirely new, and in the Solution of it (in the laſt part of Prop 23.) there is a remarkable Inſtance of the Uſefulneſs of the Method of firſt and laſt Ratio's. The 24*th* Propofition gives the Invention of the Center of Oſcillation of all Bodies ; and in the 25*th* Propofition I have given the Inveſtigation of the Center of Percuffion. It is known that this Problem is folved by the fame *Calculus* as the foregoing; wherefore it is generally thought that thefe two Centers are the fame. But that is a Miſtake, becaufe the Center of Oſcillation can be but one Point; but the Center of Percuffion may be any where in a certain Line, which this Propofition fhews how to find. There is an Error in this Propofition, which I was not fenſible of till after the Book was publiſh'd, wherefore I take this Opportunity of correcting of it. It does not affect the Reafoning by which I find the Diſtance of the Center of Percuffion from the Axis of Rotation ; but it is this, that I fuppofed the Center of Percuffion to be in the Plane paſſing thro' the Center of Gravity, and perpendicular to the Axis of Rotation : which is a Miſtake. It is corrected by the following Propofition.

PROP.

P R O P.   P R O B,

To find the Distance of the Center of Percussion from the Plane passing thro' the Center of Gravity and perpendicular to the Axis of Rotation.

S O L U T I O N.

Let the sixth Figure be suppoſed in the Plane paſſing thro' the Axis of Rotation, and in which the Center of Percuſſion is ſought.

Let $A B$ be the Axis of Rotation, $A G C$ be the Interſection of this Figure with the Plane paſſing thro' the Center of Gravity, and perpendicular to the Axis of Rotation, G be the Point whereon a Line, rais'd perpendicular to this Figure, will paſs thro' the Center of Gravity ; $B E$ be a Line parallel to $A G$ wherein is the Center of *Percuſſion*. Then to find the Diſtance $A B$, let p ſtand for an Element of the Body propoſed ſtanding perpendicularly on any point D. Draw $D C$ perpendicular to $A G C$, and $A B$ will be equal to the Summ of all the Quantities $p \times G C \times C D$ taken with their proper Signs, divided by the Body it ſelf multiplied into the Diſtance $A G$.

Having thus found the Diſtance $A B$, ſuppoſe the Plane of the Figure in *Prop. 25.* to cut the preſent Figure at right Angles in the Line $B E$, and the Center of Percuſſion will be rightly determined by that Propoſition.

The 26*th* Propoſition ſhews how to determine the Denſity of the Air at any Diſtance from the Center of the Earth ſuppoſing the Denſity always to be proportional to the compreſſing Force, and that the Power of Gravitation is reciprocally as the Diſtances from the Center of the Earth.

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The laft Propofition fhews how to find the Refraction of a Ray of Light in its paffage thro' the Atmofphere, upon the Suppofition that Light is a Body, and that the Refraction of it is caufed by the Attraction of the Bodies the Rays approach to In this Propofition there is a remarkable Inftance of the Ufefulnefs of the Method of Increments in finding the Coefficients of a Series, which according to the Values of a certain Symbol, as *n*, expreffes both all the Fluents, and all the Fluxions of a certain Quantity.

II. Ludovici Ferdinandi Marfilii *Differtatio de* Generatione Fungorum. *Rom.* 1714. 4to.

THis Author tells us that he gave his youthful Inclinations to the Study of the Mathematicks and Obfervations of Nature, under the Tuition of the celebrated *Malpigius,* and *Lelius Triumfettus* Botanick Profeffor of *Bononia* : and amongft the various Productions of Nature, his chief Delight was in the Contemplation of the fudden Growth and various forts of *Mufhrooms* which both the Earth and Trees brought forth. Of the firft Kind he obferved the greateft Number to arife in Camps, produced from the Horfes Dung, and are commonly called *Prataiouli.*

In the Years 1699 and 1700, being then in *Croatia* and *Tranfylvania,* in the Armies there, he made a large Volume of Defigns of *Fungi,* which he fent to *Triumfetti* to put in order, who added a great Number to them of fuch as he found about *Bononia* ; yet after all the moft diligent Search, he could never find them to produce any Seed either in their Gills or other Parts.

The Origin and Generation of *Mufhrooms* he fays is not eafy to demonftrate, fince both the Antients and Moderns dif-

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difagree very much about it. The late Botanifts feem to
be of three different Sentiments concerning their Produce.
Mr. *Ray*, Dr. *Sherrard*, Mr. *Doody*, *Boccone* and *Mentzelius*
having obferved fome *Mufhrooms* to have had Seed, were
of opinion others might have the fame Original. *Clufius*
and *John Baptifta Porta* had in fome alfo obferved their
Seed : Others, *viz. Sharrock* and the accurate *Malpigius*,
who could not find any Seed in them, altho' with the
Affiftance of Microfcopes, did fuppofe they might be pro-
duced by Pieces of themfelves, carried by the Winds from
place to place, as other Plants are by flips and Offsetts.

The third Opinion, which he fays moft agree in, is
that they arife from Putrefaction, or a Mixture of cer-
tain Salts, Sulphur and Earth impregnated with the Dung
of Beafts.

The *Fungus feminifer Campaniformis Mentzelii*, &c.
being the Mufhroom which firft gave the occafion of the
Opinion of their having Seed, this nice Author has ac-
curately figured and obferved, and fuppofes with others
that thefe feedlike Bodies may be the *Ovaria* of fome In-
fects ; and the rather becaufe they are fo very large in
proportion to the fmallnefs of the *Mufhroom* : and that
they had often been fowed by Dr. *Amadoes* a curious
Botanift, without any Succefs towards raifing them.
From whence he concludes thefe Bodies ought to have
another Denomination than Seed ; neither is he of the
Opinion that they are produced by parts of themfelves.

In his Divifion of *Mufhrooms* he firft treats of the *Truffles*
and their Increafe, Situation and Soyl, Colour, Taft and
Confiftence. He next proceeds to foft Mufhrooms, fuch
as he obferved in his own Garden ; which having in the
Spring been meliorated with Horfe-dung, about the mid-
dle of *June* there fprung up divers of that fort which the
Italians call *Prataiuoli*, amongft a Bed of *Lettice*. Thefe
continued till near the midft of *Auguft* before they went
off.

off. Of thefe and fome other Kinds he accurately figures·
the firft Shootings and Fibres.

His next Tribe are fuch as grow from Wood, but yet
are themfelves foft. Of thefe he obferves three Kinds;
the firft a large one in his Window, out of a piece of
Firwood which it had often rained on ; with two fmaller
forts from fome rotten Boards in his Garden. All thefe he
figures both in their natural and divided States, as alfo
Microfcopically

Treating of hard woody *Mufhrooms* (of which he alfo
gives you fome accurate Figures) he obferves they rarely
appear on the Trees, in *Germany* and *Croatia*, before they
are twenty or thirty Years old ; but moft commonly
when forty or fifty: and the Original of them he attributes
chiefly to the Rottennefs of the Wood, and fays they
generally break out in the Spring, when the Leaves begin
to fhoot. And that ufually they grow below the middle
of the Trees, and are caufe of fo much Decay in them,
that they often die in three or four Years.

It may not here be amifs to fubjoin what Dr. *Lancifius*
communicates to our Author, concerning the *Lapis Fun-
garius*, *viz.* that altho' this *Mufhroom-producer* has the
Name of a Stone, it ought not to be reckoned of that
Genus, it being really no other than a Mafs or *Congeries* of
Roots, Seeds and Juices coagulated with Earth into, as
it were, a ftony Subftance. Upon which pouring Water
and fetting it in a warm Place, it loofens its hardned Sub-
ftance ; and by mollifying its Fibres and moiftning its con-
crete Juices, out of the Cliffs and Chinks thereof the *Mufh-
rooms* fpring, as they do in other places from fimple Dung
and loofe Earth And it is alfo farther to be noted, that
when this ftony Mafs has thus yielded thefe its Offspring,
the Remains grows light, porous and decay'd, its nu-
tritive Juices being then exhaufted.

F I N I S.